CS135 Section 9.5

page 615/3 a), b), c) $f, g: \mathbb{Z} \to \mathbb{Z}$ equiv. relation?

a) $\{ (f,g) \mid f(1) = g(1) \}$

- it is reflexive, because $f(1) = f(1)$, hence $(f,f) \in R$
- it is symmetric, because if $f(1) = g(1)$, then obviously $g(1) = f(1)$, so if $(f,g) \in R$, then $(g,f) \in R$
- it is transitive, because

if $f(1) = g(1)$ and $g(1) = m(1)$, then obviously $f(1) = m(1)$

i.e. if $(f,g) \in R$ and $(g,m) \in R$, then $(f,m) \in R$

therefore it is an equivalence relation

b) $\{ (f,g) \mid f(0) = g(0) \text{ or } f(1) = g(1) \}$

- it is reflexive, since $f(0) = f(0)$ or $f(1) = f(1)$ ~~is true~~, hence $(f,f) \in R$
- it is symmetric, because obviously if $f(0) = g(0)$ or $f(1) = g(1)$, then $g(0) = f(0)$ or $g(1) = f(1)$
- but it is not transitive, because if

$$\left.\begin{array}{l} f(0) = g(0) \text{ or } f(1) = g(1) \\ \quad\text{and} \\ g(0) = m(0) \text{ or } g(1) = m(1) \end{array}\right\}$$ we cannot conclude that $f(0) = m(0)$ or $f(1) = m(1)$

what if $f(0) = g(0)$ and $g(1) = m(1)$?

- no connection between f and m can be established

Therefore the given relation is not an equivalence relation

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page 615/3 (e) $\{(f,g) \mid f(x)-g(x)=1 \quad \forall x \in \mathbb{Z}\}$

- it is not reflexive, because $f(x)-f(x)=0 \quad \forall x \in \mathbb{Z}$, so $(f,f) \notin R$.

Therefore, the relation is not an equivalence relation

page 615/9 $A \neq \emptyset \qquad f: A \to$

$R = \{ (x,y) \mid f(x)=f(y) \}$

a) Let's show that R is an equivalence relation:

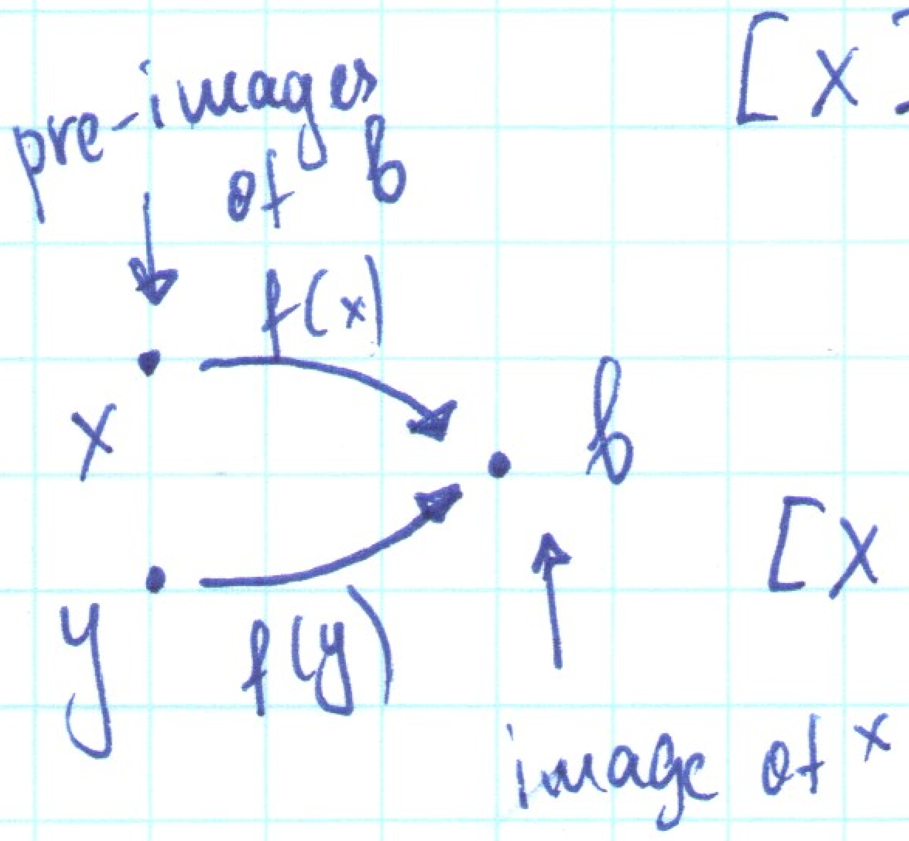
- it is reflexive, because $f(x)=f(x)$, i.e. $(x,x) \in R \quad \forall x \in A$
- it is symmetric, because if $(x,y) \in R$ it means that $f(x)=f(y)$, but $f(y)=f(x)$ also holds, so $(y,x) \in R$ as well
- it is transitive, because if $(x,y),(y,z) \in R$, then it means that $f(x)=f(y)$ and $f(y)=f(z)$. Obviously $f(x)=f(z)$, so $(x,z) \in R$ as well

we proved that R is an equivalence relation

b) the equivalence classes of R:

$[x]_R = \{ y \mid f(x)=f(y) \}$ because only pairs (x,y) s.t. $f(x)=f(y)$ belong to R

or



$[x]_R = \{ f^{-1}(b) \mid f(y)=f(x)=b \}$

all pre-images of image of x